E462 Family

University of Tripoli
Faculty of Engineering
Electrical & Electronic Engineering Department

*Homework (2)*

EE462 - Digital Control Systems

1. Prove that $\Delta E = E\Delta$, (i.e. the operators $\Delta$ & $E$ are commutative *w.r.t* multiplication).
2. Solve the following difference equations (use two methods):
   (i) $y(k+2) + 4y(k+1) + 3y(k) = 0$ , $y(0) = 0$ , $y(1) = 1$
   Recur → (ii) $(\Delta^2 - \Delta)y(k) = k$ , $y(0) = 0$ , $y(1) = 1$
   (iii) $\left(E^2 + \frac{3}{4}E + \frac{1}{8}\right)y(k) = 0$ , $y(0) = 0$ , $y(1) = 1$
   (iv) $(E^2 + 2E + 1)y(k) = 2(-1)^k$ , $y(0) = y(1) = 1$
3. Obtain the z-transform of the following functions: (do not use table)
   (i) $e(k) = k$
   (ii) $e(k) = k^2$
   (iii) $e(k) = k\,a^k$
   (iv) $e(k) = k\,\sin(ak)$
   (v) $e(t) = t\cos(\omega t)$
   (vi) $e(t) = t\,e^{-at}$
4. Determine the inverse z-transform of each $E(Z)$ below (use two methods):
   (i) $E(Z) = \dfrac{1}{(Z-1)(Z-3)}$
   (ii) $E(Z) = \dfrac{Z^2}{(Z+1)(Z+2)^2}$
   (iii) $E(Z) = \dfrac{Z^2 + 6Z + 4}{Z^2 + \frac{1}{4}Z - \frac{1}{8}}$
   (iv) $E(Z) = \dfrac{2Z^2}{(Z-1)^3}$

[Adv & disAdv of ZOH FOH]

*Good Luck...*

Relaxes sys
initial conditions = 0

* Difference operators:
① E → shifting operator.
$E[y(k)] \triangleq y(k+1)$
$E^n[y(k)] \triangleq y(k+n)$
② Δ → forward difference operator
$\Delta y(k) = y(k+1) - y(k)$
$\Delta y(k+1) = y(k+2) - y(k+1)$

Represent DE by E
$(E^n + a_{n-1}E^{n-1} + \dots + a_1 E + a_0)\,y(k) = u(k)$, in E = order

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$\dfrac{0.1065\,z + 0.0902}{(z-1)(z-0.6065)}$

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EE462 " Digital control systems"
*Fall 2019* — *Final Examine* — *2:30*

Answer all Questions

Q1.a. Find **E(z):** for $E(s) = [2(1-e^{-s})]/[s^2+0.2s]$, if [ T=1sec ]

b. A digital control system described by the difference equation;
$(E^3 - 5/6E^2 + 1/6E)\,y(k) = (0.5)^k$
(i) Define the order of D.E. , then mention the **order** of the system.
(ii) Find the T.F. and sketch the poles then comment on stability.
(iii) If I.C. are zero determine the response of the system

Q2.a. Find **c(k)** if $C(z) = z/(z^3 - 5/6\,z^2 - 1/3z + 1/6)$

b. A unity F.B. digital control system **$G_p(s) = 1/(s + 0.2)$** S/H [ T=1sec, ZOH ]
if D(z) is given by D.E. **m(k) = e(k) – 0.5 e(k -1) + m(k -1)**
(i) Determine the open loop T.F. then mention its **type.**
(ii) Determine the closed loop T.F. then calculate **the unit step response.**
(iii) Determine the **static error coefficients**, then obtain their $e_{ss}$.

Q3.a. Find **$E^*(s)$** : for $E(s) = 1/[(s^2+3s+2)]$ , [ T = 0.1 sec]

b. A **unity F.B.** digital control system has S/H [ T=0.5sec, ZOH ], D(z)=1 and
**$G_p(s) = 1/s(s+1)$** in the forward path,
(i) Calculate **ζ , $\omega_n$** and τ , then if the **(S/H)** is removed find **ζ , $\omega_n$** and τ.
(ii) Obtain the **unit step** response for the system.
(iii) Determine the **steady state error.**

Q4.a. Find **$G(j\omega_w)$.** if $G(s) = 4(1-e^{-Ts})/[s^2(s + 2)]$ , [ T = 0.1sec

b. If O.L.T.F. of digital control system with unity F.B. is given by,
$G(z) = k(z + 0.8)/[(z-1)(z-0.6)]$, with [ T = 1sec ]
(i) On rectangular paper construct the **Root Locus** and label all **the pertinent points with their gains.**
(ii) Determine the range of the **gain k for stability** by any method.
(iii) From the graph show any **gain** (if exist) which make the system to **oscillate** and its frequency of oscillation.

...... GOOD LUCK

0.75 + 0.3663j
0.834∠0.454
∠26.03
order
0.3677

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EE462 " Digital control systems"

*Fall 2018* *$1^{st}$ Examine* *1:20*

Answer all Questions

Q1.a. find **E(z)**: (i) $e(t) = 2e^{-2t}\cos 2t$ (ii) $E(s) = [2(1-e^{-s})]/[s^2+0.2s]$, [T=1sec]

b. A digital control system is modeled by;

$y(k+2) - 1.1y(k+1) + 0.3y(k) = (-0.2)^k$

if, $y(0) = 0$, $y(1) = 1$

(i) obtain the **discrete transfer function** then comment on the stability.

(ii) determine **y(k) in closed form**.

(iii) determine the $\mathbf{1^{st}}$ **five** sequences {y(k)} (use **Recursive** formula).

(iv) in z-plane, show the roots of system T.F.

Q2.a. find **c(k)** by **two** methods: $C(z) = z/(z^3 - 5/6\, z^2 - 1/3 z + 1/6)$

b. A digital control system with unity F.B. has S/H [T= 0.1 sec ,ZOH], $D(z) = (z-0.4)/(z-1)$ and $G_p(s) = 1/(s + 2)$ in the forward path.

(i) what is the system type?

(ii) Obtain the open loop transfer function.

(iii) determine the **unit step response c(k)**.

(iv) determine the static error coefficients & their $e_{ss}$ .

Q3.a. find $\mathbf{E^*(s)}$ : (i) $E(s) = 1/[(s^2+3s+2)]$ , (ii) $e(t) = 2 - 2e^{-2t}$ [ T = 0.1 sec]

b. if the open loop transfer function for closed loop system with unity F.B. is given by;

$D(z)\,G(z) = k(0.3z + 0.2)/(z^2 - 1.2z + .2)$

(i) Find the characteristic equation **Q(z).**

(ii) What is the **range of k** for stable system?

(iii) Choose a **value for k** which make the system **to oscillate**.

(iv) Choose **a value for k** for stable system with **minimum** $e_{ss}$ .

**Comment on your answers**

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EE462 " Digital control systems"

spring2018 *$1^{st}$ Examine* *1:30*

Answer all Questions

Q1.a. $r(k) = 2k \sin 2k$ find R(z)

b. A digital control system is described by;

$(E^3 + 5/6E^2 - 1/3\, E - 1/6)\, y(k) = u(k)$

(i) find the $1^{st}$ six sequences if $u(k) = (-1)^k$ and $y(0) = 0$, $y(1) = 1$, $y(2) = 1.5$

(ii) determine y(k) in closed form if u(k) is a unity discrete function & I.C. = 0

(iii) obtain the T.F. then comment on the stability by any method

(iv) sketch the roots in z-plane then show the response of each root.

Q2.a. $E(s) = (s+3)/(s^2 + 2s)$ find $[E^*(s), E(z)]$

b. A digital control system with unity F.B. has S/H [T= 1sec ,ZOH], D(z) and $G_p(s) = 1/(s + 0.2)$ in the forward path if D(z) is given by;

$m(k) = e(k) - 0.5\, e(k-1) + m(k-1)$

(i) determine the unit step response

(ii) what is the system type?

(iii) find the steady state error for all standard inputs.

Q3.a. $C(z) = 2z/(z^2 + 2/3 z + 1/9)$ find c(k)

b. A digital control system with unity F.B. has S/H [T= 0.1sec ,ZOH], D(z) and $G_p(s) = 4/(s + 4)$ in the forward path if $D(z) = K_p + (K_I T/(z-1))$

(i) find $K_I$ so that $e_{ss}$ due to a unit ramp input is 0.04 (assume stable system)

(ii) Determine the range of $K_p$ for stable system ( use $K_I$ from (i) )

Q4.a. $r(k) = 2k^2 e^{2k}$ find R(z)

b. A digital control system has a transfer function,

$Y(z)/U(z) = (2z^2 + z)/(z^3 - 1.5 z^2 + 0.66z - 0.08)$

(i) Test the stability by Jury's method.

(ii) Obtain the difference equation.

...... GOOD LUCK

Al-Fateh University — E' Department

$E^2 462$ "0101010 Control Systems"

S- 2010 — $2^{nd}$ Examine — 1:30

1. A digital control system with unity F.B. has S/H (T=0.1 sec, ZOH), D(z)=1 and $G_P(s) = 4/(s(s+2))$ in the forward path;
   - (i) Find the open loop T.F.
   - (ii) Determine the Time constant, damping ratio & the undamped natural frequency.
   - (iii) If S/H is removed determined the paramerters in part (ii).
2. A unity F.B. digital control system has S/H (T=0.1 sec, ZOH), D(z)=k & $G_P(s) = 4/(s(s+2))$ in the forward path;
   - (i) Plot the Root Locus and label all the pertinent values on the graph
   - (ii) Determine the range of k for stable system.
   - (iii) What is the value of k which make the system to oscillate and its frequency of oscillation?
   - (iv) Find the open loop T.F. G(w).
3. A digital control system has the plant frequency response $G(jW_w)$ in the given table.
   - (i) On semi-log paper draw $|G|_{db}$ & phase curves.
   - (ii) Find the system gains & their frequencies.
   - (iii) Design a phase lag controller that yields a phase margin 55 if dc gain equal to 10

Good Luck

| $\omega_w$ | $\omega$ | $\lvert G(j\omega_w)\rvert$ | $\lvert G(j\omega_w)\rvert_{dB}$ | $\angle G(j\omega_w)$ |
|---|---|---|---|---|
| 0.1 | 0.100 | 19.97513 | 26.00 | −93.14 |
| 0.2 | 0.200 | 9.95054 | 19.95 | −96.28 |
| 0.3 | 0.300 | 6.59317 | 16.38 | −99.38 |
| 0.4 | 0.399 | 4.90325 | 13.80 | −102.45 |
| 0.5 | 0.499 | 3.88102 | 11.77 | −105.46 |
| 0.6 | 0.599 | 3.19331 | 10.08 | −108.41 |
| 0.7 | 0.699 | 2.69741 | 8.61 | −111.28 |
| 0.8 | 0.799 | 2.32198 | 7.31 | −114.08 |
| 0.9 | 0.899 | 2.02741 | 6.13 | −116.78 |
| 1.0 | 0.999 | 1.78990 | 5.05 | −119.40 |
| 2.0 | 1.993 | 0.70945 | 2.98 | −140.61 |
| 3.0 | 2.977 | 0.37308 | −8.56 | −154.64 |
| 4.0 | 3.947 | 0.22743 | −12.86 | −164.43 |
| 5.0 | 4.899 | 0.15270 | −16.32 | −171.82 |
| 6.0 | 5.829 | 0.10973 | −19.19 | −177.74 |
| 7.0 | 6.733 | 0.08291 | −21.62 | −182.72 |
| 8.0 | 7.610 | 0.06511 | −23.72 | −187.04 |
| 9.0 | 8.457 | 0.05270 | −25.56 | −190.88 |
| 10.0 | 9.273 | 0.04372 | −27.18 | −194.33 |
| 20.0 | 15.708 | 0.01403 | −37.05 | −217.40 |
| 30.0 | 19.656 | 0.00798 | −41.96 | −229.64 |
| 40.0 | 22.143 | 0.00558 | −45.07 | −236.77 |

University of Tripoli / Faculty of Eng. — $E^3$ Depart.

EE462 " Digital control systems"

Fall 2019 — $1^{st}$ Examine — 1:30

Answer all Questions

Q1.a. find E(z); $e(t) = 3e^{-t}\cos t$

b. A digital control system has a transfer function,
$C(z)/R(z) = (z^2 + z)/(z^3 - 1.5z^2 + 0.66z - 0.08)$
- (i) Obtain the unit step response. (use Recurrence method)
- (ii) Test the system stability. (use Jury's)

Q2. a. if $Q(z) = z^3 + z^2 + z - 0.5 = 0$ Is the system stable? (Justify the answer).

b. A digital control system is modeled by;
$y(k+2) - 0.7y(k+1) + 0.1y(k) = (-0.1)^k$ if, y(0) =0, y(1) =1
- (i) obtain the discrete transfer function.
- (ii) determine the response y(k) in closed form.
- (iii) determine the $1^{st}$ five sequences {y(k)}. (use Recursive formula)
- (iv) in z-plane, show the roots of system T.F. then comment on stability.

Q3.a. find c(k); $C(z) = z/(z^3 - 5/6z^2 - 1/3z + 1/6)$

b. A digital control system with unity F.B. has S/H [T= 0.1 sec ,ZOH],
$D(z) = (z-0.4)/(z-1)$ and $G_p(s) = 2/(s+2)$ in the forward path.
- (i) Obtain the open loop transfer function. what is the system type?
- (ii) determine & sketch the unit step response c(k).
- (iii) determine the static error coefficients & their steady error.

Q4.a. find $E^*(s)$ ; $E(s) = 1/[s(s^2+3s+2)]$

b. A unity FB digital control system has $D(z)=k$ & $G_p(s) = 0.36/[s(s+0.6)]$ with S/H (T=0.5 , ZOH) find ; (i) characteristic equation Q(z). (ii) the range of K for stable system. (iii) value of K for marginally stability.

...... GOOD LUCK

Tripoli University. Faculty of Engineering
Electrical & Electronic Engineering Department
EE 462 "Digital Control Systems"

Fall 2012 $2^{nd}$ Examine 1:30

Answer all Questions

1. A unity FB digital control system has S with T=0.2s. D(z)= (z-0.4)/(z-1), ZOH and $G_p(s) = 1/(s+1)$ in the forward path;
   (i) Find the open loop transfer function. Then mention its type.
   (ii) Determine the response c(k) if the input is a unit step.
   (iii) Determine $e_{ss}$ for all standard inputs.

2. A digital control system with unity FB has S/H (T=0.5s, ZOH), D(z)=1 and $G_p(s) = 1/s(s+1)$ in the forward path;
   (i) Determine ξ, $\omega_n$ and time constant.
   (ii) If S/H is removed find ξ, $\omega_n$ and τ for the analog system.
   (iii) What is the effect of sampling?

3. An open transfer function of FB digital control system;
   $G(w) = -0.76(w-1)/(1.24w^2+0.76w)$, if $\omega_w$ changes (0 to ∞)
   (i) Plot the Bode diagram magnitude and phase.
   (ii) What are the crossover frequencies ($\omega_{wc}$, $\omega_{w\pi}$)?
   (iii) Find GM and PM, then comment on the stability.

4. A digital control system has an open loop transfer function;
   $D(z)G(z) = K(0.0132z+0.0115)/(z^2-1.646z+0.6703)$
   (i) Convert the the OLTF to the W-plane if T=2s.
   (ii) Use Routh-Hurwitz criterion to determine the range of K for stability.
   (iii) Find part (ii) by using Jury's test. What is the value of K for marginally stable system?

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EE462 " Digital control systems"

*Fall 2018* *$2^{nd}$ Examine* *1:30*

Answer all Questions

Q1. A **unity F.B.** digital control system has [S/H T=0.5s, ZOH], D(z)=1 and $G_p(s) = 1/s(s+1)$ in the forward path,
   (i) determine the **open** and **closed** loop transfer functions.
   (ii) determine the damping ratio(ζ) , the undamped natural Frequency($\omega_n$) and the time constant(τ).
   (iii) if the (S/H) is removed determine ζ, $\omega_n$ and τ
   (iv) form (ii) & (iii) what is the effect of sampling on system stability.
   (v) determine the **static error coefficients** and their $e_{ss}$.

Q2. A digital control system with **unity F.B.** has the open loop transfer function
   **G(z) = k(0.321z+0.197)/[(z-1)(z-0.223)]**, with T = 1sec
   if gain k changes from zero to infinity.
   (i) Construct the **root locus** and **label** all the **pertinent points** on the graph with their **gains.** (use the rectangular paper)
   (ii) determine the range of gain k for stability (if exist).
   (iii) what is the gain and frequency of **oscillation for marginally** stable system? (if any)
   (iv) comment on the stability of the system.

Q3. A digital control system with unity F.B. has an **open loop** transfer function,
   $G(w) = [0.76(1-w)]/[(0.76w+1.24w^2)]$
   if $\omega_w$ changes (0 to ∞)
   (i) On **semi log** paper draw the Bode diagram magnitude & phase.
   (ii) what are the gain & phase margins and their crossover frequencies?
   (iii) Is the system stable according to the obtained margins in (ii) comment?
   (iv) check the stability by using Routh-Hurwitz criterion.

GOOD LUCK

$G(z) = \frac{0.01873 z + 0.01752}{(z-1)(z-0.81873)}$

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EE462 " Digital control systems"

spring 2019 — 2$^{nd}$ Examine — 1:30

Answer all Questions

Q1. A **unity F.B.** digital control system has [S/H, T= 0.2sec, ZOH], D(z) = k and $G_p(s)$ = **1/s(s+1)** in the forward path,

(i) determine the open and closed loop transfer functions.

(ii) By using bilinear transformation[ $z=\frac{1+\frac{T}{2}w}{1-\frac{T}{2}w}$ ] obtain **Q(W).**

(iii) determine the range of k for stable system (if any), (use R-H criterion).

(iv) determine the static error coefficients and their $e_{ss}$.

Q2. A digital control system with unity F.B. has an **open loop** transfer function,

G(z) = **[0.758(z+1)]/[z$^2$ - 1.242z + 0.242)]** ,

with T = 2 sec, if $\omega_w$ changes (0 to ∞).

(i) write down the equations of magnitude in db & phase in degree.

(ii) Plot the Bode diagram $|G(j\omega_w)|_{db}$ and $\angle G(j\omega_w)$ w.r.t $\omega_w$.

(iii) Obtain from the diagram GM & PM as well as $\omega_{wc}$ and $\omega_{w\pi}$.

(iv) Is the system stable according to the obtained gain margins in (ii)? Comment on your answer.

Q3. An open loop transfer function of a digital control system with **unity F.B.** is given by G(z) = **k(0.321z+0.197)/[z$^2$-1.223z+0.223)],**

with T = 1sec , if the **gain k** changes (0 to ∞).

(i) Construct the **root locus** on rectangular paper show all steps.

(ii) determine and **label** all **pertinent points** on the graph with their **gains** (break in & away and the intersection of locus with unit circle).

(iii) what is the gain and frequency of **oscillation for marginally** stable system? (if any)

(iv) comment on the stability of the system.

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z = -3.720

z = 0.19265

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spring2018 — 2$^{nd}$ Examine — 1:30

Answer all Questions

Q.1. A **unity F.B.** digital control system has [S/H T=0.1s, ZOH] and $G_p(s)$ = **4/s(s+2)** in the forward path,

(i) determine the open and closed loop transfer functions.

(ii) what is the system type?

(iii) determine the static error coefficients and their $e_{ss}$.

(iv) calculate the unit step response for the system.

Q2. A digital control system with **unity F.B.** has [S/H [T= 0.1s1 ,ZOH], D(z)=k and $G_p(s)$ = **2/[s(s + 2)]** in the forward path,

(i) determine range of k for stable system use Jury and Routh tests.

(ii) what is the value of k which make the system to oscillate & its frequency of oscillation?

Q3. A digital control system with **unity F.B.** has the forward path transfer function

G(z) = **k(z+0.8)/[(z-1))(z-0.6)],**

if **gain k** changes from zero to infinity.

(i) Construct the **root locus** and **label** all the **pertinent points** on the graph with their **gains.**

(ii) determine the range of gain k for stability (if exist).

(iii) what is the frequency of **oscillation for marginally** stable system?

Q4. A digital control system has an **open loop** transfer function,

G(w) = **[1.516(1-w)]/[2(0.758w+1.242w$^2$)]**

if $\omega_w$ changes (0 to ∞)

(i) On **semi log** paper draw the Bode diagram magnitude & phase.

(ii) what are the gain & phase margins and their crossover frequencies?

(iii) Is the system stable according to the obtained margins in (ii) comment?

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